Pre-Calculus **Chapter 8 Review** **Name:**_____
March 2014

In #1-2, a point P $(r,\ \theta)$ is given in polar coordinates. Give <u>two</u> other polar representations of the point, one with $r < 0$ and one with $r > 0$.

1. $\left(3, \frac{3\pi}{4}\right)$

2. $(-2, 9\pi)$

In #3-4, convert the polar coordinates to rectangular coordinates $(x,\ y)$.

3. $\left(-4, \frac{5\pi}{3}\right)$

4. $\left(2, -\frac{5\pi}{2}\right)$

In #5-6, convert the rectangular coordinates to polar coordinates with $r > 0$ and $0 \le \theta < 2\pi$.

5. $(\sqrt{8}, \sqrt{8})$

6. $(1, -2)$

In #7-8, a complex number is given. Write the number in polar form $[r(\cos\theta + i\sin\theta)]$ with argument between 0 and 2π.

7. $5 - 5i$

8. $-1 + \sqrt{3}i$

In #9-10, find the indicated power and roots. Find exact values if possible (make sure answers are in $a + bi$ form).

9. $\left(-3 - 3\sqrt{3}i\right)^4$

10. Find the 4^{th} roots of $1 - i$

In #11-15, sketch the vector indicated.

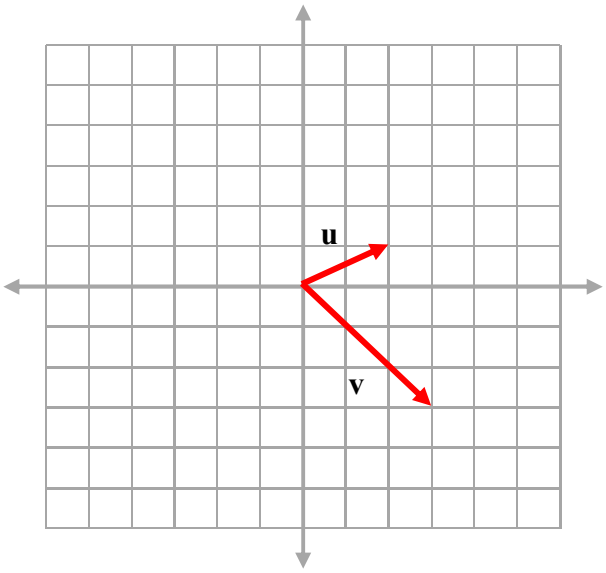
11. 3**u**

12. **– u**

13. -2**v**

14. **v – u**

15. **u + v**



In #16-17, express the vector with initial point P and terminal point Q in component form.

16. P(1, 1); Q(9, 7)

17. P(-1, 3); Q(-6, -1)

In #18-21, find $-2\mathbf{u},\ \mathbf{u}+\mathbf{v},\ 2\mathbf{v}-\mathbf{u},\ |\mathbf{u}|,\ |\mathbf{v}|$, ***and*** $|\mathbf{u}-\mathbf{v}|$ ***for the given vectors*** $\mathbf{u}$ ***and*** $\mathbf{v}$.

18. $\mathbf{u}=\langle -2,\ 5\rangle,\ \mathbf{v}=\langle 2,\ -8\rangle$

19. $\mathbf{u}=\mathbf{i}+\mathbf{j},\ \mathbf{v}=\mathbf{i}-\mathbf{j}$

20. $\mathbf{u}=-2\mathbf{i}+3\mathbf{j},\ \mathbf{v}=\mathbf{i}-2\mathbf{j}$

21. $\mathbf{u}=\langle -6,\ 6\rangle,\ \mathbf{v}=\langle -2,\ -1\rangle$

In #22-25, find the horizontal and vertical components of the vector with the given length and direction. Write your answer in component form AND in terms of $\mathbf{i}$ ***and*** $\mathbf{j}$.

22. $|\mathbf{v}|=40,\ \theta=\frac{2\pi}{3}$

23. $|\mathbf{v}|=\sqrt{3},\ \theta=300°$

24. $|\mathbf{v}|=5,\ \theta=225°$

25. $|\mathbf{v}|=2,\ \theta=\frac{\pi}{2}$

In #26-29, find the magnitude and direction of the vector. Write your answer in radians.

26. $\langle -\frac{\sqrt{2}}{2},\ -\frac{\sqrt{2}}{2} \rangle$

27. $\mathbf{v} = \mathbf{i} + \mathbf{j}$

28. $\mathbf{u} = -\mathbf{i} - \sqrt{3}\mathbf{j}$

29. $\langle -4\sqrt{3},\ 4 \rangle$

In #30-33, find $\mathbf{u} \cdot \mathbf{v}$ for the given vectors u and v.

30. $\mathbf{u} = 5\mathbf{i} + \mathbf{j}$, $\mathbf{v} = 6\mathbf{i} - 2\mathbf{j}$

31. $\mathbf{u} = \langle 3,\ -5 \rangle$, $\mathbf{v} = \langle -2,\ 7 \rangle$

32. $\mathbf{u} = \langle 0,\ 4 \rangle$, $\mathbf{v} = \langle 10,\ -1 \rangle$

33. $\mathbf{u} = -3\mathbf{i} - 2\mathbf{j}$, $\mathbf{v} = 2\mathbf{j}$

In #34-37, find the angle between u and v to the nearest degree.

34. $\mathbf{u} = \mathbf{i} + \mathbf{j}$, $\mathbf{v} = \mathbf{i} - \mathbf{j}$

35. $\mathbf{u} = \langle -6,\ 6 \rangle$, $\mathbf{v} = \langle 1,\ -1 \rangle$

36. $\mathbf{u} = \langle 3,\ 4 \rangle$, $\mathbf{v} = \langle 2,\ 1 \rangle$

37. $\mathbf{u} = \mathbf{i} + \sqrt{3}\mathbf{j}$, $\mathbf{v} = -\sqrt{3}\mathbf{i} + 2\mathbf{j}$

In #38-39, determine whether the vectors are orthogonal.

38. $\mathbf{u} = \langle 5,\ 3\rangle,\ \ \mathbf{v} = \langle -2\ ,\ 6\rangle$

39. $\mathbf{u} = \mathbf{i} - \mathbf{j},\ \ \mathbf{v} = \mathbf{i} + \mathbf{j}$

In #40-43, find the indicated quantity, given $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j},\ \ \mathbf{v} = -4\mathbf{i} + \mathbf{j},\ \ \mathbf{w} = 6\mathbf{j}.$

40. $\mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

41. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$

42. $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v})$

43. $(\mathbf{u} \cdot \mathbf{v})(\mathbf{u} \cdot \mathbf{w})$

In #44-45, find the work done by the force **F** ***in moving an object from P to Q.***

44. $\mathbf{F} = 400\mathbf{i} + 50\mathbf{j};\ \ \mathrm{P}(-1,\ 1),\ \ \mathrm{Q}(200,\ 1)$

45. $\mathbf{F} = -4\mathbf{i} + 20\mathbf{j}\ ;\ \mathrm{P}(0,\ 10),\ \ \mathrm{Q}(5,\ 25)$

46. The force $\mathbf{F} = 5\mathbf{i} - 6\mathbf{j}$ moves an object 10 feet along the x-axis in the positive direction. Find the work done if the unit of force is the pound.

47. A lawn mower is pushed a distance of 100 feet along a horizontal path by a constant force of 60 lb. The handle of the lawn mower is held at an angle of $45°$ from the horizontal. Find the work done.

Answers:

1. $\left(3,\ \frac{11\pi}{4}\right);\ \left(-3,\ \frac{7\pi}{4}\right)$
$\left(3,-\frac{5\pi}{4}\right);\ \left(-3,-\frac{\pi}{4}\right)$
two examples of each

2. $(2,\ 0\pi);\ (-2,\ \pi)$
$(2,\ 2\pi);\ (-2,\ 3\pi)$
two examples of each

3. $\left(-2,\ 2\sqrt{3}\right)$

4. $(0,\ -2)$

5. $\left(4,\ \frac{\pi}{4}\right)$

6. $\left(\sqrt{5},\ \tan^{-1}(-2)\right)$

7. $z = 5\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

8. $z = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

9. $-648 - 648i\sqrt{3}$

10. $.213 + 1.07i, -1.07 + .213i$
$-.213 - 1.07i,\ 1.07 - .213i$

11-15. Will be shown in class, and will be posted online

16. $\langle 8,\ 6\rangle$

17. $\langle -5,\ -4\rangle$

18. $\langle 4,\ -10\rangle,\ \langle 0,\ -3\rangle,\ \langle 6,\ -21\rangle,\ \sqrt{29},\ 2\sqrt{17},\ \sqrt{185}$

19. $-2\mathbf{i} - 2\mathbf{j},\ 2\mathbf{i},\ \mathbf{i} - 3\mathbf{j}, \sqrt{2},\ \sqrt{2},\ 2$

20. $4\mathbf{i} - 6\mathbf{j},\ -\mathbf{i} + \mathbf{j},\ 4\mathbf{i} - 7\mathbf{j},\ \sqrt{13},\ \sqrt{5},\ \sqrt{34}$

21. $\langle 12,\ -12\rangle,\ \langle -8,\ 5\rangle,\ \langle 2,\ -8\rangle,\ 6\sqrt{2},\ \sqrt{5},\ \sqrt{65}$

22. $\langle -20,\ 20\sqrt{3}\rangle,\ -20\mathbf{i} + 20\sqrt{3}\mathbf{j}$

23. $\langle \frac{\sqrt{3}}{2},\ -\frac{3}{2}\rangle,\ \frac{\sqrt{3}}{2}\mathbf{i} - \frac{3}{2}\mathbf{j}$

24. $\langle -\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\rangle,\ -\frac{5\sqrt{2}}{2}\mathbf{i} - \frac{5\sqrt{2}}{2}\mathbf{j}$

25. $\langle 0,\ 2\rangle,\ 2\mathbf{j}$

26. $1,\ \frac{5\pi}{4}$

27. $\sqrt{2},\ \frac{\pi}{4}$

28. $2,\ \frac{4\pi}{3}$

29. $8,\ \frac{5\pi}{6}$

30. 28

31. -41

32. -4

33. -4

34. $90°$

35. $180°$

36. $27°$

37. $71°$

38. Not orthogonal

39. Orthogonal

40. -26

41. -26

42. -4

43. 168

44. 80,400 ft-lb

45. 280 ft-lb

46. 50 ft-lb

47. $3000\sqrt{2}$ ft-lb